**Cover Page**

**AMS 2940**

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**Title of Lab Report: Optimizing Production to Maximize Profit**

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**Submitted to: Dr. Hisham Hussein**

1. **Introduction:**

* **Problem definition**

We are investigating Saudi Food Company, who have contributed greatly to a variety of industries including milk, ice cream, coffee and tea and many more. The company has also produced a variety of juices with different flavors such as apple, mango, pineapple etc.

This problem comprises of 10 juices with various raw materials such as sodium, carbohydrate, sugar, dietary and water given to create the juices and these two points will be our key data for formulating our linear programming problem, as well as solving and analyzing its results.

The following tables give the distribution of raw materials, cost and selling price per unit for each juice type.

TABLE I

Amount of Raw Materials

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Flavors** | **Sodium (kg)** | **Carbohydrate (kg)** | **Dietary (kg)** | **Sugar (kg)** | **Water (liter)** |
| Pineapple | 0.01 | 0.012 | 0.005 | 0.012 | 6.822 |
| Orange | 0.01 | 0.008 | 0.005 | 0.008 | 7.552 |
| Fruit Cocktail | 0.01 | 0.015 | 0.01 | 0.015 | 4.824 |
| Mango | 0.005 | 0.015 | 0.005 | 0.015 | 7.621 |
| Apple | 0.005 | 0.012 | 0.01 | 0.012 | 6.539 |
| Grape | 0.01 | 0.014 | 0.005 | 0.014 | 7.602 |
| Pomegranate | 0.005 | 0.010 | 0.01 | 0.013 | 6.12 |
| Strawberries | 0.01 | 0.008 | 0.005 | 0.010 | 7.055 |
| Guava | 0.005 | 0.012 | 0.01 | 0.012 | 7.508 |
| Banana | 0.01 | 0.012 | 0.005 | 0.008 | 7.051 |

TABLE II

Cost & Price per units

|  |  |  |  |
| --- | --- | --- | --- |
| **Products** | **Average Cost (SR)** | **Average Selling (SR)** | **Profit (SR)** |
| Pineapple | 1 | 2.19 | 1.19 |
| Orange | 1 | 2.51 | 1.51 |
| Fruit Cocktail | 1 | 2.51 | 1.51 |
| Mango | 1 | 2.19 | 1.19 |
| Apple | 1 | 2.51 | 1.51 |
| Grape | 1 | 2.41 | 1.41 |
| Pomegranate | 1 | 2.10 | 1.10 |
| Strawberries | 1 | 2.19 | 1.19 |
| Guava | 1 | 2.15 | 1.15 |
| Banana | 1 | 2.08 | 1.08 |

TABLE III

Quantity of Raw Materials in Stock

|  |  |
| --- | --- |
| **Raw Materials** | **Quantity Available** |
| Sodium (kg) | 4,037 |
| Carbohydrate (kg) | 23,651 |
| Dietary (kg) | 4,651 |
| Sugar (kg) | 46,712 |
| Water (liter) | 16,376,630 |

* **Importance of solving the problem**

It is important to solve this problem to illustrate how many of each type of juice need to be manufactured in order to maximize profit for Saudi Food Company. Furthermore, it would show how much of each raw ingredient is required to produce the optimal amounts, thus resulting in reduced costs of raw materials for the company.

Moreover, they would like to have an efficient and organized production schedule by taking advantage of sensitivity analysis and the solution to this linear programming problem.

1. **Linear Programming Formulation:**

* **Decision Variables**

X1 – Pineapple

X2 – Orange

X3 – Fruit Cocktail

X4 – Mango

X5 – Apple

X6 – Grape

X7 – Pomegranate

X8 – Strawberry

X9 – Guava

X10 – Banana

* **Objective Function**

ZMax = 1.19X1 + 1.51X2 + 1.51X3 + 1.19X4 + 1.51X5 + 1.41X6 + 1.10X7 + 1.19X8 + 1.15X9 + 1.08X10

* **Constraints**
* 0.01X1 + 0.01X2 + 0.01X3 + 0.005X4 + 0.005X5 +

0.01X6 + 0.005X7 + 0.01X8 + 0.005X9 + 0.01X10 ≤ 4,037 (sodium constraint)

* 0.012X1 + 0.008X2 + 0.015X3 + 0.015X4 + 0.012X5 +

0.014X6 + 0.010X7 + 0.08X8 + 0.012X9 + 0.012X10 ≤ 23,651 (carbohydrate constraint)

* 0.005X1 + 0.005X2 + 0.01X3 + 0.005X4 + 0.01X5 +

0.005X6 + 0.01X7 + 0.005X8 + 0.01X9 + 0.005X10 ≤ 4,651 (dietary constraint)

* 0.012X1 + 0.008X2 + 0.015X3 + 0.015X4 + 0.012X5 +

0.014X6 + 0.013X7 + 0.010X8 + 0.012X9 + 0.008X10 ≤ 46,012 (sugar constraint)

* 6.822X1 + 7.552X2 + 4.824X3 + 7.671X4 + 6.539X5 +

7.602X6 + 6.12X7 + 7.055X8 + 7.508X9 + 7.051X10 ≤ 16,376,630 (water constraint)

* Xi ≥ 0 for i = 1,…,10 (nonnegativity constraint)

1. **Method of solving problem and reasoning behind it:**

The simplex method is going to be used to solve this linear programming problem because it consists of large number of decision variables and constraints. Moreover, it will help illustrate more clearly the basic and non-basic variables involved in this problem.

1. **The solution to the problem using Mathematica:**

* **Primal Problem**

ZMax = 1.19X1 + 1.51X2 + 1.51X3 + 1.19X4 + 1.51X5 + 1.41X6 + 1.10X7 + 1.19X8 + 1.15X9 + 1.08X10

**s.t** 0.01X1 + 0.01X2 + 0.01X3 + 0.005X4 + 0.005X5 +

0.01X6 + 0.005X7 + 0.01X8 + 0.005X9 + 0.01X10 ≤ 4,037 (sodium constraint)

0.012X1 + 0.008X2 + 0.015X3 + 0.015X4 + 0.012X5 +

0.014X6 + 0.010X7 + 0.08X8 + 0.012X9 + 0.012X10 ≤ 23,651 (carbohydrate constraint)

0.005X1 + 0.005X2 + 0.01X3 + 0.005X4 + 0.01X5 +

0.005X6 + 0.01X7 + 0.005X8 + 0.01X9 + 0.005X10 ≤ 4,651 (dietary constraint)

0.012X1 + 0.008X2 + 0.015X3 + 0.015X4 + 0.012X5 +

0.014X6 + 0.013X7 + 0.010X8 + 0.012X9 + 0.008X10 ≤ 46,012 (sugar constraint)

6.822X1 + 7.552X2 + 4.824X3 + 7.671X4 + 6.539X5 +

7.602X6 + 6.12X7 + 7.055X8 + 7.508X9 + 7.051X10 ≤ 16,376,630 (water constraint)

Xi ≥ 0 for i = 1,…,10 (nonnegativity constraint)

ZMax = SR 1,000,102

When ->

X1 = 0X2 = 0X3 = 0X4 = 684,600

X5 = 122,800X6 = 0X7 = 0X8 = 0

X9 = 0X10 = 0S1 = 0S2 = 11,908.400391

S3 = 0S4 = 34,269.398438S5 = 10,322,074

* **Dual Problem:**

We constructed the dual problem of our problem by taking the right-hand side of the constraints as our new coefficients of the new objective function. The new problem is now a minimization problem with 5 variables and 10 constraints constructed by taking the transpose of the constraints and placing the respective new variables into them. Our new right-hand sides are the coefficients of the original objective function and the inequality signs of constraints are given by the inequality signs of the variables in the original problem. Furthermore, the inequality of the new variables are given by flipping the signs of the original constraints.

ZMin = 4037U1 + 23651U­­2 + 4651U3 + 46012U4 + 16376630U5

**s.t** 0.01U1 + 0.012U2 + 0.005U3 + 0.012U4 + 6.822U5 ≥ 1.19

0.01U1 + 0.008U2 + 0.005U3 + 0.008U4 + 7.552U5 ≥1.51

0.01U1 + 0.015U2 + 0.01U3 + 0.015U4 + 4.824U5 ≥1.51

0.005U1 + 0.015U2 + 0.005U3 + 0.015U4 + 7.671U5 ≥1.19

0.005U1 + 0.012U2 + 0.01U3 + 0.012U4 + 6.539U5 ≥1.51

0.01U1 + 0.014U2 + 0.005U3 + 0.014U4 + 7.602U5 ≥1.41

0.005U1 + 0.010U2 + 0.01U3 + 0.013U4 + 6.12U5 ≥1.10

0.01U1 + 0.08U2 + 0.005U3 + 0.010U4 + 7.055U5 ≥1.19

0.005U1 + 0.012U2 + 0.01U3 + 0.012U4 + 7.508U5 ≥1.15

0.01U1 + 0.012U2 + 0.005U3 + 0.008U4 + 7.051U5 ≥1.08

Ui ≥ 0 for i = 1,…,10 (nonnegativity constraint)

ZMin = SR -1,000,102

When ->

U1 = 174U2 = 0 U3 = 64U4 = 0

U5 = 0S1 = 0.87S2 = 0.55S3 = 0.87

S4 = 0S5 = 0 S6 = 0.65S7 = 0.41

S8 = 0.87S9 = 0.36S10 = 0.98

1. **Sensitivity Analysis:**

* Optimality Range of Obj. Function Coefficients

In conducting our Sensitivity Analysis I decided to illustrate the allowable increase or decrease in the current objective function coefficient as a range to maintain optimality.

∞ ≤ X1 ≤ 2.0481

∞ ≤ X2 ≤ 0.5500

∞ ≤ X­3 ≤ 0.8700

0.183333 ≤ X4 ≤ 0.3200

0.000000 ≤ X5 ≤ 0.5500

∞ ≤ X6 ≤ 0.6500

∞ ≤ X7 ≤ 0.4100

∞ ≤ X8 ≤ 0.8700

∞ ≤ X9 ≤ 0.0000

∞ ≤ X10 ≤ 0.9800

* Feasibility Range of RHS

Regarding the right hand side of the constraints we also constructed a feasibility range for the RHS to allow easy readability and an allowable increase or decrease that would not affect the linear programming problem.

1,711.500000 ≤ B1 ≤ 614.000000

11,908.400391 ≤ B2 ≤ ∞

614.000000 ≤ B3 ≤ 3,423.000000

34,269.398438 ≤ B4 ≤ ∞

10,322,074.000000 ≤ B5 ≤ ∞

1. **Interpretation of results:**

* Primal Problem

Beginning with the primal problem, we obtained a maximum value of SR 1,000,102. The simplex method used a specific style of solution named the parametric LP method to solve the Linear Programming Problem. Furthermore, the number of basic variables in the final tableau is 5 including X4, X5, S2, S3, and S4. However, the number of non-basic variables is 10. Based on the final tableau, the maximum value is dependent on the value of X4 and X5.

* Dual Problem

As for the dual problem, we had to reformulate the problem to keep it identical to the primal problem but in dual form, by alternating the inequality signs of new constraints, new variables. Moreover, we changed the coefficients of the new variables in the objective function. Using the simplex method, we obtained the minimum value of SR -1,000,102. However to reach a feasible outcome we multiplied this result by -1 to obtain the maximum value of the Linear Programming Problem with U1, U3, S1, S2, S3, S6, S7, S8, S9, and S10 as basic variables to maximize the objective function.

* Sensitivity Analysis

With respect to the sensitivity analysis of the problem, I conducted an algebraic sensitivity analysis to this linear programming problem to individually find the allowable range of increase and decrease for each respective coefficient of the variables in the objective function for optimality range. Furthermore, the feasibility range was found for each right-hand side of each constraint of the primal problem.

Based on our results, the optimality ranges for our coefficients is quite flexible as some variables’ coefficients can decrease to infinity and most are limited by a certain increase in value to maintain optimality. As for the right-hand side of our constraints, all of them are easily flexible to decrease by large values, and some can be increased by infinity – still maintaining feasibility. This illustrates a probability that the coefficients of the objective function hold greater weight towards maintaining the optimality of the problem.

**References**

Kriri, Q. (2018, April 02). Use of linear programming for optimal production in a production line in SAUDI Food Co. Retrieved April 16, 2021, from <https://publications.waset.org/10008981/use-of-linear-programming-for-optimal-production-in-a-production-line-in-saudi-food-co>